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LETTER TO THE EDITOR

Crack growth in a plastic medium

M P López-Sancho†, F Guinea† and E Louis‡

† Instituto de Ciencia de Materials (CSIC), Serrano 144, 28006 Madrid, Spain
‡ Departamento de Fisica, Facultad de Químicas, Universidad de Alicante, Apdo 99, 03080 Alicante, Spain

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Abstract. Crack growth in a medium with both elastic and plastic behaviour is simulated by means of a discrete model. The system interpolates between a purely elastic case, in which cracks develop a fractal structure, similar to that found in other growth models, and purely elastic deformations, where crack propagation can be reduced to the Eden model.

The nature of the processes which determine crack formation and growth in materials is far from understood (see, for instance, Englman and Jaeger (1986)). Simple models, based on analogies with growth mechanisms proposed for other physical systems, may prove useful and give some insight into this complicated phenomenon. In this letter, we will extend previous studies by two of us (Louis and Guinea 1987) on a model of *mechanical breakdown* for purely elastic materials, to include plastic effects as well. The crack patterns generated are presented and analysed numerically. From a theoretical point of view, it is worth remarking that the model to be studied includes, as opposite limits, two situations of interest in the physics of growth processes: in the absence of plasticity, we have the mechanical breakdown model already mentioned, which is the vectorial counterpart of diffusion-limited aggregation (Witten and Sander 1983) and dielectric breakdown (Nimeyer *et al* 1984); with no elastic effects, our system reduces itself to the Eden model of cluster growth (Eden 1961). Our model is a two-dimensional, triangular lattice of springs which simulates, in the continuum limit, an isotropic material. The response of each spring under an applied stress is linear. The springs near the surface of a crack have a finite probability of failing, which is linearly proportional to their deformation. Once they fail, they continue to exert a force on the remaining bonds, which is a fixed fraction, α, of the stress they stored at the moment of failure, as shown in figure 1. When $\alpha = 0$, this scheme is equivalent to

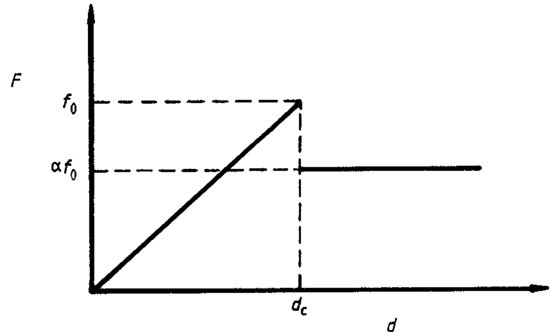

Figure 1. Force exerted by a given spring as function of its deformation. d_c gives the maximum deformation before failure and plastic behaviour.

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the removal of the bond which failed, i.e. the model of mechanical breakdown. This case describes a very brittle material, perfectly elastic up to a threshold where it loses completely its ability to support stresses. For $\alpha = 1$, the response of each bond is in accordance with that predicted by the theory of a purely plastic material.

To analyse crack propagation, an external hydrostatic pressure is applied on the boundaries of the lattice, to create a finite stress distribution in its interior. Then, a bond in the centre of the sample is made to fail. This modifies the local stress distribution, and the position of the lattice nodes are changed to achieve the new equilibrium configuration. The stresses in the bonds near the one which failed are calculated, and one of them is chosen randomly, with probability proportional to the absolute value of its deformation; this bond is again made to fail, and the size of the crack grows. We iterate this procedure until the surface of the crack is close enough to the edge of the lattice (usually 10–12 bond lengths, for triangular lattices of side 120–140).

When $\alpha = 0$ we recover the case previously studied (Louis and Guinea 1987) of mechanical breakdown (see also Meakin *et al* 1988). The crack patterns have a fractal dimension of approximately 1.65, and the overall shape is similar to DLA aggregates, which is the scalar analogue of this model. The case $\alpha = 1$ can be understood by a simple picture. The bonds are in equilibrium prior to the formation of the crack. As the force exerted by each failed bond *remains exactly the same* as it was before failure, the lattice never deviates from equilibrium. Thus the stress distribution is always the same and is fixed by the initial boundary conditions. Hydrostatic pressure induces homogeneous stresses within the material, so all bonds are subjected to the same deformation. The probability for any given bond to fail is equal to that for any other. All bonds at the surface of the crack have the same growth probability; this is the definition of the Eden model of cluster growth (Eden 1961). The resulting shapes have dimensions equal to the space dimension in which they are embedded, in our case $D = 2$, although the surface can have a complex structure (Freche *et al* 1985). Changes in the value of α smoothly interpolate between these limits.

Results for $\alpha = 0, 0.25, 0.5, 0.75$ and 1 are shown in figure 2. The patterns always have a very homogeneous structure, and can be well characterised by their fractal dimension D, which varies between 1.65 and 2, as expected. We use a linear fit in a log-log plot to define D, with very good correlations (see figure 3).

In the continuum limit, the lattice is replaced by an elastic medium with Lamé coefficients which satisfy $\lambda/\mu = 1$. The internal stresses are determined by the applied pressure and by the boundary conditions at the edges of the crack. We can define them in terms of local coordinates, given by the normal and parallel directions to the crack surface. The inner portion of the crack does not completely lose its ability to sustain stresses, and it exerts forces on the elastic part which has not failed. These forces are a finite fraction of those existing before failure; a simple continuum limit which satisfies this criterion can be obtained by assuming that the rate of change of the forces acting across a given surface element, ds, is a constant fraction of the force itself:

$$\frac{\partial \ln(F_i)}{\partial n} = \frac{1-\alpha}{\alpha} \qquad i = x, y. \tag{1}$$

When $\alpha = 0$ this equation can only be satisfied if $F_i = 0$, $i = x, y$, i.e. the boundary condition near the edge of a void in an elastic medium, in accordance with the mechanical breakdown model. In the opposite limit, $\alpha = 1$, we obtain equations similar

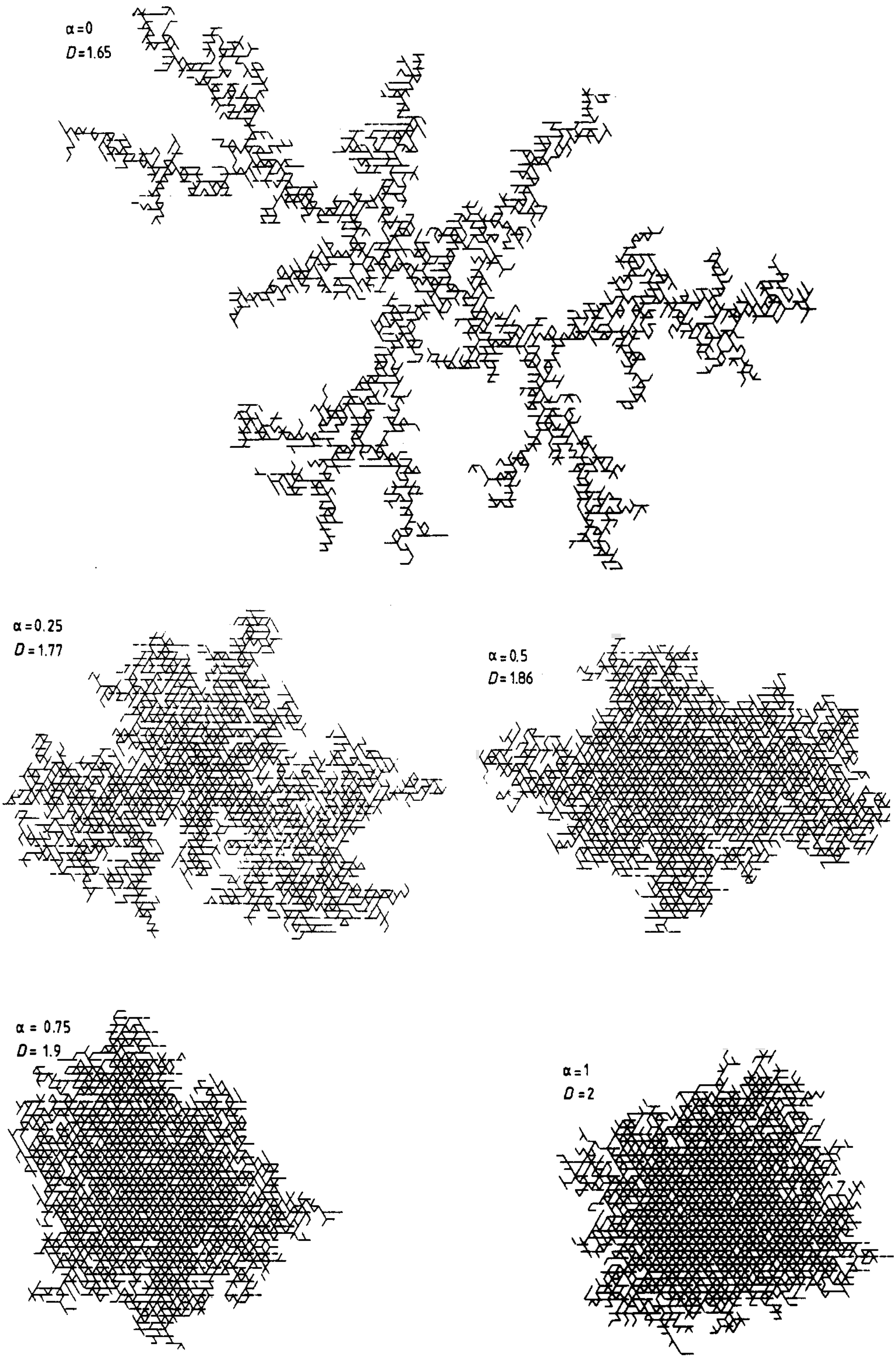

Figure 2. Crack patterns for different values of α.

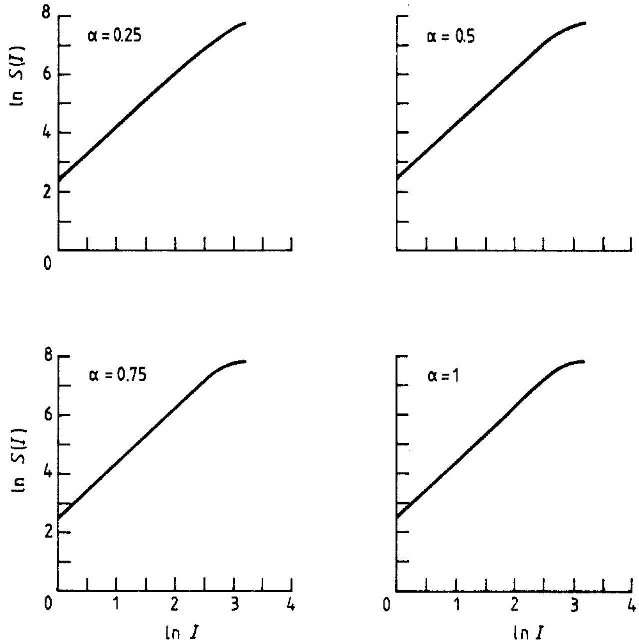

Figure 3. Log-log plots of the number of failed bonds as function of distance to the centre of the pattern for the realisations shown in figure 2 (the case $\alpha = 0$ is excluded).

to the equilibrium conditions for a volume element of an elastic medium (Landau and Lifschitz 1966); in particular, the initial stress distribution, which is constant in space, is always a solution. Thus, the equilibrium is never disturbed and the stresses remain unchanged throughout the calculations. The growth probability is equal at all points of the edges of the crack and we recover the Eden model. In terms of the components of the stress tensor, projected along the local normal and parallel directions to the surface of the fracture zone, equation (1) can be written as

$$\begin{aligned} \frac{\partial \ln(\sigma_{nn})}{\partial n} &= \frac{1-\alpha}{\alpha} \\ \frac{\partial \ln(\sigma_{np})}{\partial n} &= \frac{1-\alpha}{\alpha}. \end{aligned} \qquad (2)$$

To complete the continuum equations, the velocity of growth of a point of the surface is proportional to the *tangential* stresses acting on it:

$$\nu_n \propto \sigma_{pp}. \qquad (3)$$

This equation resembles closely the conventional DLA model and the mechanical breakdown model; on the other hand, it underlines the difference between the present calculation and other schemes used to simulate more dilute aggregates and dendritic growth (Navas *et al* 1988, Nittman and Stanley 1986). In the latter cases, the velocity of growth is made to be proportional to a higher power of the stress.

In conclusion, we present a model of crack formation in elastoplastic materials. The similarities and differences with other growth models of current interest are discussed. A continuum model is proposed to describe the main features of the calculations, and facilitate analytical studies. While the present scheme is still too simple to describe the rich phenomenology of fracture in real materials, we think that it provides a starting point to develop further its theoretical understanding.

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